Appendix 3

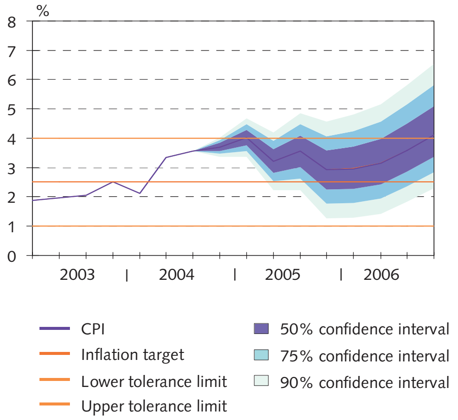
Uncertainty in the Central Bank's inflation forecast

Since the Central Bank of Iceland moved on to an inflation target at the beginning of 2001 it has published inflation forecasts with a two-year horizon. These forecasts play a key role in monetary policy decisions. The inflation forecast is always based on the technical assumption that the policy interest rate remains unchanged over the horizon. The forecast is used to gauge whether the current policy rate is sufficient to maintain a rate of inflation as close as possible to the Central Bank's target of 2½%. A substantial deviation in the outlook generally calls for a change in the policy rate. However, there is no mechanical connection between the forecast and monetary policy decisions.

Inflation forecasts are subject to uncertainty which increases over the horizon. In making interest rate decisions, the Central Bank takes into account not only the main inflation forecast, but also its probability distribution. To underline this and also to inform the public and markets about the risk profile, the Central Bank publishes confidence intervals for each forecast, i.e. the ranges within which inflation will end up with a 50%, 75% and 90% probability. The probability distribution is represented in the form of a fan chart with increasingly dark lines as the interval narrows.[1] An assessment of the main asymmetric uncertainties also accompanies the forecast. This approach contributes to a more focused analysis of various factors that may impact the forecast and underlines their importance in forecast preparations. An example of the fan chart is Chart 1, which shows the Central Bank's forecast for Q4/2004. It also shows that there was an upside risk to the inflation forecast at end-2006.

The following is a closer examination of methods for estimating the probability distribution for the forecast and the balance of risks.

Chart 1
Bank of Iceland inflation forecast 2004/4
In *Monetary Bulletin* 2004/4



Probability distribution in the inflation forecast

Uncertainty in the Central Bank's inflation forecasts is estimated on the basis of historical data on its forecasting errors one and two years ahead.[2] However, since the degree of uncertainty may vary, historical data do not necessarily give a clear indication of future uncertainty. For each forecast, therefore, an assessment is made of whether the degree of uncertainty calculated from historical data should be scaled up or down. Likewise, the forecasting risk can be to the upside or the downside, i.e. when inflation one or two years ahead is considered

1. This implies a 10% probability that inflation will end up outside the shaded area of the chart.
2. Since the Central Bank began publishing quarterly inflation forecasts two years ahead in *Monetary Bulletin* 2001/2, the standard deviation of the forecasting error over that horizon has been 1.1%. The standard deviation of the forecasting error one year ahead has been slightly higher at 1.2%. The assessment is still based on a relatively few observations but a more reliable measure of the standard deviation of the forecasting error should be obtained over time. A study of the forecasting errors in the Bank's inflation forecasts is published every year in *Monetary Bulletin*, most recently in *Monetary Bulletin* 2004/2.

more likely to be greater than the main forecast (which is regarded as the most probable value) or lower.

The risk profile for the inflation forecast is based on methods developed by the Bank of England and Sveriges Riksbank (Britton et al., 1998, and Blix and Sellin, 1998), which also allow an estimation of skewed distributions.

A two-piece normal distribution is used, see Johnson et al. (1994):

$$(1) \quad f(x) = \left(\frac{2}{\sqrt{1/(1-\gamma)} + \sqrt{1/(1+\gamma)}} \right) \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{(x-\mu)^2}{2\sigma^2} \left[1 + \gamma \left(\frac{x-\mu}{|x-\mu|} \right) \right] \right\}$$

where $f(x)$ is the density function, μ is the mode of the probability distribution (i.e. the value that maximises the density function) and σ is the standard deviation of the composite density function.

The parameter γ measures the skewness of the probability distribution and lies in the range -1 to +1. The asymmetric uncertainty can then be calculated from γ , measured as the deviation of the mean from the mode of the distribution, which is expressed with φ:

$$(2) \quad \varphi = (m - \mu) = \sqrt{2/\pi} \left(\frac{\sigma}{\sqrt{1-\gamma}} - \frac{\sigma}{\sqrt{1+\gamma}} \right) = \sqrt{2/\pi} (\sigma_2 - \sigma_1)$$

where m is the mean of the distribution and σ_1 and σ_2 are the standard deviation of the two parts of the composite probability distribution. Standard deviation σ_1 therefore measures the standard deviation of the distribution to the left of μ and σ_2 to the right of μ.[3] If $\gamma > 0$ the distribution is skewed upwards ($m > \mu$) to leave a larger part of it to the right of the mode, i.e. $\sigma_2 > \sigma_1$. Conversely, if $\gamma < 0$ the distribution is skewed downwards ($m < \mu$) to leave a larger part to the left of the mode, i.e. $\sigma_1 > \sigma_2$. For a conventional symmetric normal distribution, $\gamma = 0$ with $\sigma_1 = \sigma_2$ and $m = \mu$. The density function in equation (1) simplifies to:

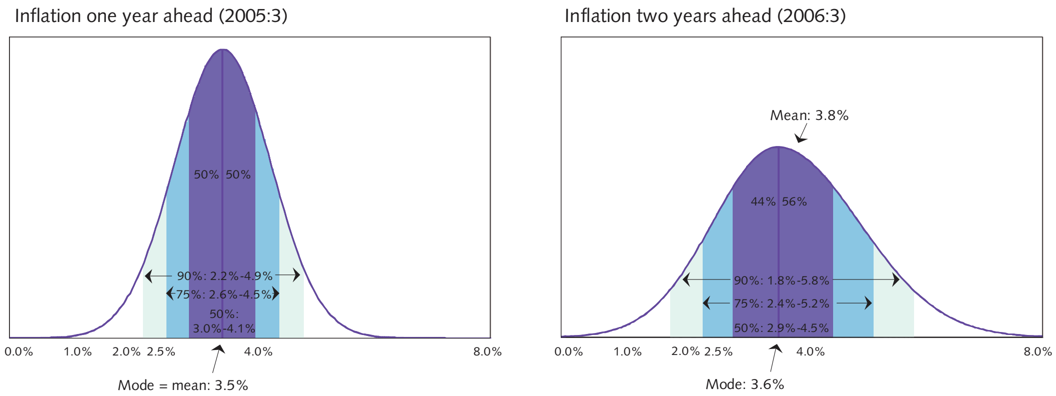
$$(3) \quad g(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2} \right)$$

Chart 2 shows the probability distribution one and two years ahead for the inflation forecast that was published in *Monetary Bulletin* 2004/4 (i.e. forecast inflation in Q3/2005 and Q3/2006). The best way to understand the presentation of the Central Bank's inflation forecast is to examine Charts 1 and 2 together. In effect, the probability distribution for inflation is calculated separately for each of the nine quarters that the Bank forecasts, as shown in Chart 2. Chart 1 then presents a simple bird's-eye view of these nine probability distributions.

The width of the probability distribution reflects the risks in the forecast: the wider they are, the greater the uncertainty about how developments will unfold. This is shown, for example, by the fact that the probability distribution two years ahead is much wider than the

3. These are in effect two conventional normal distributions measured with their respective standard deviations rescaled to be continuous in the mode with the integral below the area equal to 1.

Chart 2
Probability distribution one and two years ahead for the Central Bank of Iceland inflation forecast 2004/4



distribution one year ahead, because the uncertainty increases over the forecast horizon. Since the area below the curve must always equal 1, increased uncertainty is also reflected in a lower and flatter curve. The risk profile is ultimately reflected in the shape of the distribution: symmetric uncertainty is reflected in symmetric probability distribution, but the distribution will be skewed if the estimated risk is greater in either direction.

In the Central Bank's forecast in December 2004, the risk was considered symmetric one year ahead but on the upside two years ahead.[4] Inflation was forecast at 3.5% one year ahead which, since the balance of risks was symmetric, also corresponds to the mean of the forecast. Two years ahead, however, the most probable rate of inflation was considered to be 3.6%. Since that distribution is skewed to the upside, however, the mean of the forecast was 3.8%. Accordingly, 56% of the probability distribution lies above the mode and only 44% below it. It was considered fairly unlikely that the inflation target would be attained over the forecast horizon based on the policy interest rate at that time. One manifestation is that the probability of inflation in the range 2-3% one and two years ahead was only 20%.

Uncertainty assessment

An assessment of uncertainties in the inflation forecast attempts to give a forward-looking view of the risks to the forecast, not a mechanical extrapolation of past forecasting errors. It examines the underlying factors in the development of inflation and assesses whether the uncertainty is greater or less than is implied by historical forecasting errors or fluctuations in these values. Whether the risk is to the upside or downside is also estimated. Factors at work include exogenous economic developments (e.g. exports, oil prices and the general level of import prices), domestic demand (e.g. private consumption, investment, the public sector, imports, wage developments and the output gap) and financial market developments (e.g. the exchange rate and equity prices).

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4. The assessment of forecast uncertainty thus allows the probability of inflation falling within a given range over the next two years to be calculated, cf. Charts 1 and 2 which show, for example, a 50% probability that inflation two years ahead would be in the range just below 3-4.5%, and Table 7 in *Monetary Bulletin* 2004/4 which shows a 57% probability that it would be in the range 1-4%, i.e. within the tolerance limits.

A calculation of uncertainties in the inflation forecast therefore simply examines fluctuations in these factors and the standard deviation of the forecast is their weighted mean, where N is the number of subfactors:

$$(4) \qquad \sigma = \left(\frac{\sum_{i=1}^{N} \beta_i h_i \sigma_i}{\sum_{i=1}^{N} \beta_i \sigma_i} \right) \omega$$

where ω is the historical standard deviation in the Central Bank's inflation forecasting errors, σ_i is the forecasting error in the respective factor and β_i measures the impact of each subfactor on inflation one and two years ahead. The parameter h_i is the scaling factor for that value and is greater than 1 if the uncertainty about the factor is considered greater than historical forecasting errors would imply (and thus $\sigma > \omega$) but lower than 1 if the uncertainty is considered less (and thus $\sigma < \omega$). To give an example, the Central Bank has considered the uncertainty in its recent inflation forecasts to be generally lower than historical forecasting errors would imply, due to the impact of forecasting errors in 2001 when the exchange rate framework was changed and a substantial depreciation of the króna went hand in hand with a temporary rise in the inflation rate.[5]

For each new forecast, an estimation is made of the main asymmetric uncertainties, i.e. those which will result in either higher or lower inflation if they materialise. This yields an estimate of the asymmetry of each subfactor, γ_i, and thus of the asymmetry of the probability distribution for the inflation forecast as a whole as:[6]

$$(5) \qquad \varphi = \sum_{i=1}^{N} \beta_i (m_i - \mu_i) = \sum_{i=1}^{N} \beta_i \varphi_i = \sqrt{2/\pi} \sum_{i=1}^{N} \beta_i h_i \sigma_i \left(\frac{1}{\sqrt{1-\gamma_i}} - \frac{1}{\sqrt{1+\gamma_i}} \right)$$

For example, in the most recent forecast, the risk connected with exchange rate and wage developments, the wealth and credit effect on private consumption, and doubts about an adequately tight fiscal stance was considered to be on the upside (i.e. γ_i for those factors exceeded 0), while asset prices were more likely to weaken further ahead (i.e. γ_i for this factor was less than 0). All told, therefore, the risk was symmetric one year ahead but to the upside two years ahead.

References

Blix, M. and P. Sellin (1998), Uncertainty bands for inflation forecasting, *Sveriges Riksbank Working Paper*, no. 65.

Britton, E., P. Fisher and J. Whitley (1998), The Inflation Report projections: Understanding the Fan Chart, *Bank of England Quarterly Bulletin*, February 1998.

Johnson, N. L., S. Kotz and N. Balakrishna (1994), *Continuous Univariate Distributions*, vol. 1, New York: Wiley.

5. The exchange rate will probably remain one of the main uncertainties in inflation developments and large-scale changes in it cannot be ruled out in the future. However, the pass-through of exchange rate fluctuations to inflation may have decreased after the exit from the fixed exchange rate regime.

6. Strictly speaking, equations (4) and (5) describe the uncertainty in the forecast one year ahead. Uncertainty two years ahead will also be affected by the risks one year ahead.