Appendix 1

Forward interest rates and their application in Central Bank analysis

The Central Bank of Iceland's main macroeconomic and inflation forecast is based on the technical assumption of an unchanged policy interest rate and exchange rate from the day of the forecast. The reason is that the Central Bank is interested in seeing a picture of future economic developments if the policy rate and exchange rate are unchanged. In light of the main forecast the policy rate can then be set so that the target can be attained. Such a forecast may present a misleading picture of future economic developments because it is unlikely that these important technical assumptions will hold. The assumption of an unchanged interest rate may have sweeping consequences when inflation diverges substantially from target or when a boom is expected and the need for tighter monetary policy is obvious. Expectations about tighter monetary policy affect the yield curve. In *Monetary Bulletin* 2004/4 the Central Bank introduced an alternative scenario to its main forecast where the interest rate and exchange rate were allowed to change. In that scenario, the policy interest rate was allowed to evolve according to forward interest rates.

Forward interest rates can be extracted from the term structure, i.e. they are implied in the spot interest rates at any given time. Suppose that a bond is traded on Icelandic Stock Exchange for 91.573 kr. with a face value of 100 kr. and maturity of 2 years. The yield of the bond is then $r_2 = 4.5\%$ which is also the 2-year spot interest rate. By computing yields of more financial instruments we can form a term structure of interest rates. The term structure not only informs us about interest rates from today until the maturity date, but also implies expected interest rates in the future. Suppose that an investor can invest in the aforementioned bond for two years or another bond for 1 year which he can renew for a further year in one year's time with a known yield. To prevent arbitrage, both investment opportunities must be equal, which means that the following must hold:

$$(3) \qquad \frac{100}{(1+r_2)^2} = \frac{100}{(1+r_1)(1+f)}$$

Where r_1 denotes the 1-year spot rate, r_2 denotes the 2-year spot rate and f denotes implied forward rates for 1 year in one year's time. If the 1-year spot rate according to the term structure is $r_1 = 3.5\%$, then the implied forward rate can be computed as:

$$(4) \qquad f = \frac{(1+r_2)^2}{(1+r_1)} - 1 = \frac{1{,}045^2}{1{,}035} - 1 = 5.51\%$$

When interpreting the forward rate it must be borne in mind that it can contain a forward term premium due to the unpredictability of future interest rates. This has not been investigated for Iceland, but Svensson (1994) points out that, although frequently tested, the forward term premium has widely been found to be negligible.

Continuous term structure is not visible on the market, but we can view the discrete connection between individual financial instruments and time. This information provides the building blocks for a continuous function which describes the term structure. There is more than one known procedure to estimate the continuous term structure. Initially McCulloch (1971, 1975) used cubic spline procedures to bridge the discount function. The discount function can be transformed to present a spot rate curve and then the implied forward rate curve can be computed in the same way as above. The cubic spline procedure has the disadvantage (especially at the longest maturity) that estimates of forward rates can be rather unstable. Consequently, other procedures have become more popular, such as the Nelson and Siegel (1987) procedure along with Svensson's (1994) extensions. Their procedure is to estimate the following equation for the forward rate:

$$(5) \qquad f(m;\beta) = \beta_0 + \beta_1 \exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1}\exp\left(-\frac{m}{\tau_1}\right) + \beta_3 \frac{m}{\tau_2}\exp\left(-\frac{m}{\tau_2}\right)$$

where f denotes the forward rate as a function of time to settlement m and the parameters β_0, β_1, β_2, β_3, τ_1 and τ_2.[1] The equation consists of four components (Nelson and Siegel had only three components but Svensson added the last part). The first part is a constant β_0. The second part is a monotonically decreasing (or increasing) part $\beta_1 \exp(-m/\tau_1)$ which works as an asymptote to ensure that the longest maturity of the forward term structure approaches $\beta_0 + \beta_1$ (which has to be positive to ensure a positive interest rate). The last two parts of the equation make it possible to have a hump-shaped yield curve. The Nelson Siegel approach enables one hump but the Svensson extension makes it possible to add another.

To make the equation functional, the parameters have to be estimated. The spot interest rate curve can be derived by integrating the function and then the discount function is easily derived as well. The parameters of the equation are estimated by either minimising price errors or yield errors. Minimising price errors involves minimising the squared difference between estimated prices from the discount function and observed prices of the financial instruments. Since the yield often has a limited effect on price at short maturity, minimising price errors can result in inaccurate estimation of the yield in that part of the yield curve. The Central Bank therefore minimises yields, which involves minimising the squared difference between estimated yields

1. The implied forward rate is continuously compounded. It is easier to use the continuously compounded rate when estimating the function, but it can be converted afterwards to, for example, weekly compounded rates by using $r_d = d(\exp(r_c/d) - 1)$ where r_d denotes weekly compounded rates, r_c denotes continuously compounded rates and d denotes number of days.

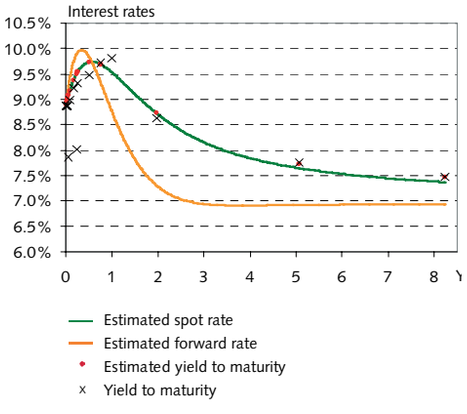
and observed yields. When the parameters have been estimated we have a continuous function which describes the term structure at a given time.

Before we estimate the term structure we have to decide what information we use as building blocks for it. For a central bank on an inflation target it is particularly interesting to obtain information from financial markets on expectations about the future policy interest rate over a horizon of roughly two years. The policy interest rate applies to repurchase agreements, which makes it interesting to investigate the one-week implied forward rate. Treasury bills and Government bonds are usually used as a basis for the (non-indexed) term structure. These bonds are selected since they are more actively traded than others. They also have a minimum default risk and therefore a minimum risk premium, unlike corporate bonds, for which the market in Iceland is too inactive to be useful in term structure estimation. When estimating the term structure the Central Bank has also used money market interest rates.[2] Money market interest rates have a disadvantage, since they are not the same financial instruments as bonds and can differ from the interest rates on Treasury bills and Government bonds even though their maturity is similar.[3] The advantage, however, is that they provide much more information on the term structure. Lately there has been an inconsistency between interest rates on Treasury bills and in the money market which can be traced to a discrepancy between supply and demand for Treasury bills.

Chart 1 shows the estimation for 1-week yield curves on February 23, 2005. The estimated spot rate rises faster than money market interest rates imply – to more than 9.7% in little more than half a year compared with just under 9.5% in the money market for the same maturity. Treasury bill interest rates differ with a yield to maturity of around 8% even though they mature after a fairly short time. Basing term structure only on Treasury bills and Government bonds would have given a quite different picture because interest rates on Treasury bills have been quite low recently. However, since there are only two Treasury bills they do not have much effect on the estimation. Estimated yield to maturity is the same as estimated spot rates for all instruments except those carrying coupons, namely Government bonds maturing in approximately 5 and 8 years. Estimated forward rates rise faster than estimated spot rates and peak at just under 10% in 3 months. After that they fall rapidly to 7% in 3 years' time.

Information regarding the evolution of interest rates is important for the Central Bank. Interest rate changes affect domestic

Chart 1
Estimated term structure of interest rates on February 23, 2005



Source: Central Bank of Iceland.

2. In *Monetary Bulletin* 2004/4 the implied forward rate was based on Treasury bills and Government bonds. Since then the methodology has been revised and interest rates in the money market are now also included in term structure estimations. The methodology is still being revised.
3. Money market interest rates are simple (flat) and have to be adjusted to an effective rate in order to be comparable with Treasury notes and Government bonds. Day count rules also differ. In the money market the actual/360 rule applies but the 30/360 rule applies for Treasury notes and Government bills.

demand and thereby inflation. Due to the lag in the pass-through, it is important for the Central Bank to be forward-looking in its monetary decisions. By monitoring the market the Central Bank can extract information about the future economic situation and the monetary stance. Disregarding the forward term premium, the implied forward rate can be interpreted as the market's forecast for the 1-week interest rate in the future, comparable to the policy interest rate. This enables the Central Bank to monitor market expectations of future monetary policy measures. The implied forward rate is also used for the macroeconomic forecast in which interest rates and exchange rates are allowed to change, as was done in *Monetary Bulletin* 2004/4.

If the term structure for indexed debt instruments is computed as well, the Bank can assess market inflation expectations, which is the difference between the real and nominal forward rate. Market inflation expectations are an indicator of the Central Bank's credibility in its decisions. If expectations are not close to the policy rate, this indicates that the monetary stance lacks credibility.

Sources:

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