

Debt that grows in excess of income cannot be sustainable. Sooner or later, such behaviour must cease, if not due to the borrower's prudence, then due to the lender's actions. This applies to public sector finances as well. If public sector finances are to be sustainable, the ratio of public debt to GDP must either remain constant or decrease.

In examining whether the public sector's operations and balance sheet can be deemed sustainable, it is necessary to consider two factors: the interest burden on the debt, and the cyclically adjusted primary balance of public sector.<sup>1</sup> In order to gain a better understanding of this, it is useful to express the public sector budget constraint as:

$$(1) \quad B_t = B_{t-1} + i_t B_{t-1} - D_t$$

where  $B_{t-1}$  is net debt at the beginning of the period  $t$ ,  $B_t$  is net debt at the end of the period,  $i_t$  is the nominal interest rate during the period, and  $D_t$  is the primary balance. The accounting relationship in Equation (1) can be rewritten as a proportion of GDP in nominal terms,  $P_t Y_t$ , where  $P_t$  is the price level and  $Y_t$  is real GDP (where  $\Delta$  represents change):<sup>2</sup>

$$(2) \quad \Delta b_t = \rho_t b_{t-1} - d_t$$

where  $b_t = B_t / (P_t Y_t)$ ,  $d_t = D_t / (P_t Y_t)$  and  $\rho_t$  represents the difference between accrued real interest on debt and output growth and is given as  $\rho_t = (1 + i_t) / [(1 + \pi_t)(1 + \gamma_t)] - 1 \approx i_t - \pi_t - \gamma_t$  when  $i_t$ ,  $\pi_t$  (inflation) and  $\gamma_t$  (output growth) are relatively low values.

Equation (2) therefore states that changes in the debt ratio correspond to the real interest burden in excess of output growth (i.e., the product of the real interest rate in excess of output growth and the debt ratio) less the primary balance as a proportion of GDP.

From Equation (2) it can be seen that, in order to prevent the debt ratio from rising, the cyclically adjusted primary balance as a proportion of GDP should at least equal the real interest burden over and above output growth. If the objective is to reduce the debt ratio, the primary balance must therefore be greater than the real interest expense over and above output growth.

Equation (2), however, sets no real constraints on a government's decisions on taxation or expenditure apart from any existing limits on the government's access to credit or funding through seigniorage. In order to see more clearly how public sector debt restricts governmental decisions, it is necessary to examine the intertemporal budget constraint. This can be done by solving Equation (2) over time. By assuming that, in equilibrium, long-term real interest rates are constant and output growth equals the constant growth rate of potential output, then  $\rho = \rho_t$  and:

$$(3) \quad (1 + \rho) b_{t-1} = \sum_{j=0}^{\infty} (1 + \rho)^{-j} d_{t+j} + \lim_{j \rightarrow \infty} (1 + \rho)^{-j} b_{t+j}$$

Assuming that the public sector cannot accumulate unlimited debt, it is said that the revenue and expenditure plans of the public sector fulfil the so-called NoPonzi condition, and the latter term on the right side of Equation (3) equals zero. In that instance, the intertemporal budget constraint satisfies:

1. The term cyclically adjusted primary balance refers to the budget balance excluding interest payments, after adjusting for the effects of the business cycle. In the context of governments' and central banks' consolidated balance sheets,  $D_t$  reflects conventional tax levies and public expenditures, as well as changes in central banks' base money.

2. Where  $(1 + i_t) B_{t-1} / P_t Y_t$  can be expressed as  $(B_{t-1} / P_{t-1} Y_{t-1})(1 + i_t) / [(1 + \pi_t)(1 + \gamma_t)]$ .

## Box V-1

### Sustainability of public sector debt

$$(4) \quad (1 + \rho)b_{t-1} = \sum_{j=0}^{\infty} (1 + \rho)^{-j} d_{t+j}$$

Equation (4) states simply that if the government is indebted ( $b > 0$ ), the present value of the primary balance with the discount rate  $1/(1 + \rho)$  must be high enough to cover interest payments and principal on the debt. As debt and interest payments rise and output growth declines, the government must raise taxes or reduce expenditures, either in the present or in the future.

#### Financial ratios of sustainable public sector finance

As can be seen from Equation (2), the primary balance as a proportion of GDP must equal the net interest burden as a proportion of GDP in order to maintain a constant debt ratio; that is:

$$(5) \quad \rho_t b_{t-1} = d_t$$

If it is assumed, for example, that the debt ratio remains unchanged at 125% of GDP and that equilibrium output growth is 3.2% (which is consistent with the properties of the Central Bank's macroeconomic model), the interest burden as a proportion of GDP is determined by government debt in the manner shown in Table 1.

Table 1 Debt ratio 125% of GDP, 3.2% equilibrium output growth

Interest burden % of GDP	Real interest rate	Interest level (real interest – output growth)
-0.75	2.60	-0.60
-0.50	2.80	-0.40
-0.25	3.00	-0.20
0.00	3.20	0.00
0.25	3.40	0.20
0.50	3.60	0.40
0.75	3.80	0.60

For example, if the objective is to reduce the debt ratio from 125% of GDP to 60% of GDP in 15 years' time and to pay the debt as though it were an annuity loan, it can be shown that debt service as a proportion of GDP is determined by potential output growth and the real interest rate terms that are offered (see Table 2).

Table 2 Debt ratio drops from 125% to 60% in 15 years

Debt service as % of GDP			Real interest rate
1.5% output growth	2.5% output growth	3.2% output growth	
5.30	4.43	3.79	2.60
5.56	4.61	3.97	2.80
5.75	4.80	4.15	3.00
5.94	4.98	4.33	3.20
6.13	5.17	4.52	3.40
6.33	5.36	4.70	3.60
6.52	5.55	4.89	3.80

Therefore, the question of whether tax increases or expenditure cuts will be necessary depends to a considerable degree on what credit terms are offered and whether – and if so, how quickly – the debt is to be reduced. Other things being equal, however, it will be necessary to increase the primary balance surplus if the debt

ratio rises as anticipated, given the obligations that will fall on the Treasury in the years to come. Given a potential output growth rate of 3.2% and a real interest rate of 3.6%, the cyclically adjusted primary balance must improve by  $\frac{1}{2}$  percentage point if the debt ratio increases to 125%, assuming a constant debt ratio (see Table 1). On the other hand, if the objective is to reduce the debt ratio by half in the next 15 years, the primary balance must improve by nearly  $4\frac{1}{2}$  percentage points each year (that is, slightly more than  $4\frac{1}{2}$  percentage points in Table 2 less the 0.1% primary balance based on the balance that would have been necessary prior to the financial crisis).